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The spin content of the proton

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Abstract. Considerable progress has been made in our knowledge of the spin distribution within the proton. The recently measured limits on polarized gluons in the proton suggest polarized gluons contribute very modestly to the proton spin. We will show that a modern, relativistic and chirally symmetric description of the nucleon structure naturally explain the current proton spin data. Most of the “missing" spin is carried by confined quark and antiquarks' angular momentum.

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Introduction

During the past two decades there has been a concerted level of activity mapping the distribution of spin (and angular momentum) onto the quarks and gluons that compose the nucleon. This effort was sparked by the discovery by the European Muon Collaboration (EMC) of a proton “spin crisis” [1]. EMC observed that the valence quarks carried only a very small fraction Σ of the proton spin [1]. The published measurement of the fraction $\Sigma \simeq 14 \pm 9 \pm 21\%$, indicated that it could possibly be equal to zero whereas the Ellis-Jaffe sum rule, based on the non-relativistic quark model (NRQM), predicts that $\Sigma = 1$. The unexpectedly small EMC value for Σ generated a tremendous level of theoretical and experimental activity. Theoretically several well-known aspects of hadron structure were explored [2, 3, 4] but none could generate such a small value for Σ. It was however quickly realized that the famous $U(1)$ axial anomaly could strongly influence the value of Σ and that the proton might contain a large quantity of polarized glue, see e.g., Refs. [5, 6, 7, 8] for a mathematically elegant formulation of this possible contribution to the proton spin. In addition, and in contrast to the NRQM treatment, since the u and d quarks in the proton behave relativistically, their angular momentums will contribute to the spin content of the proton. Schematically the proton spin content can be written as

$$\frac{1}{2} \quad = \quad \frac{1}{2}\Sigma + \Delta_{Glue} + L_z$$

The very recent experimental measurements at CERN, DESY, JLAB, RHIC and SLAC have shown that Δ_{Glue} gives (at best) a small contribution to the proton spin [9]. Furthermore, the accuracy of the measured Σ-value has increased and we now know that the sum of the quark helicities in the proton is about 1/3,

$$\Sigma = 0.33 \pm 0.03 \pm 0.05 \,,$$

which is considerably higher than the initial EMC suggestion. That polarized glue is not the explanation for the spin problem leads us to focus again on the suggestions which were based on physics that is more familiar to those modeling non-perturbative QCD. In particular, we [10] suggest that most of the missing spin of the proton must be carried as orbital angular momentum by the quarks and anti-quarks.

Before explaining the key physics which, when combined, appear to provide a natural explanation of Σ, in more detail, see e.g. Ref. [10], we present a quick summary of these phenomenologically well-established physics “factors". They are:

- The relativistic motion of the confined valence quarks
- The virtual excitation of anti-quarks in low-lying p-states generated by the one-gluon-exchange hyperfine interaction – in nuclear physics terms this is called “an exchange current correction"
- The pion cloud of the nucleon’s “quark core".

These three pieces of physics, tested in many independent ways (see below), all have the effect of converting quark spin to orbital angular momentum. As explained in more detail below (and in a recent publication [10]) the first reduces the spin by about one third, the second yields a reduction by an amount of order 0.15, and the third gives a multiplicative reduction by a factor of order 0.7. Combining these physics effects reduces the fraction of the proton spin carried by its quarks to about one third, i.e. $\Sigma = (1-0.35-0.15)\cdot 0.7 = 0.35$. One important recent new observation, based on a chiral analysis of data from lattice QCD [11], suggests very strongly that the pion cloud contributes very little to the Δ-N mass splitting. This new finding provides the justification for combining the corrections to the spin sum arising from one-gluon-exchange to that from the pion cloud. We now present some details of these three major reduction factors, which lead to the small value of Σ.

Relativistic valence quark motion

This effect on Σ was understood at the time of the EMC discovery. A spin-up, very light quark in an s-state, moving in a confined space, has a lower Dirac component in p-wave. The angular momentum coupling is such that for this component the spin is preferably down and reduces the “spin content" of the valence quarks. In the bag model, for example, where the massless quark’s ground state energy equals $\Omega/R \simeq 2.043/R$, the reduction factor $B = \Omega/3(\Omega-1) \simeq 0.65$. The same factor reduces the value of g_A from 5/3 to $\simeq 1.09$ in a bag model and this value changes little if one uses typical light quark current masses. (In the discussion section we will briefly indicate how our model leads to a realistic g_A value $\simeq 1.27$.) The quark energy, Ω/R is determined by the bag confinement condition that the quark current out of the spherical bag cavity of radius R is zero, i.e. in Dirac’s notation $\hat{r}\cdot\vec{j} = i\hat{r}\cdot\psi^{\dagger}\vec{\alpha}\psi = 0$ for $r = R$. Even in more modern relativistic models, where quark confinement is simulated by forbidding on-shell propagation through proper-time regularization, the reduction factor is very similar – e.g., in Ref. [12] $\Delta u + \Delta d$ is 0.67. The relativistic motion transfers roughly 35% of the nucleon spin from quark spin to valence quark orbital angular momentum.

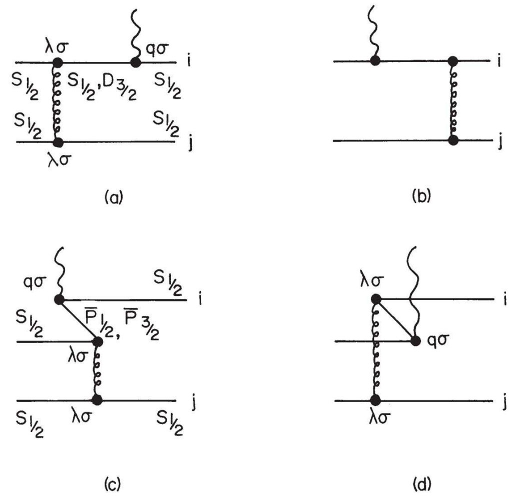

FIGURE 1. Illustration of the quark-quark hyperfine contributions which involve excited intermediate quark states. In the figures the external probe (top vertical wavy line) couples to the i'th quark which interacts with the second j'th quark via the effective confined gluon exchange. The intermediate quark propagator is evaluated as a sum over confined quark modes. Figs. (a) and (b) show the three-quark intermediate states, and (c) and (d) the one anti-quark and four quarks intermediate states. The mode sum converges rapidly and the lowest anti-quark $P_{1/2}$ and $P_{3/2}$ modes dominate the mode-sum [14].

The one-gluon-exchange hyperfine interaction

It is well established that the spin-spin interaction between quarks in a baryon, arising from the exchange of a single gluon, explains a major part of the mass difference between the octet and decuplet baryons – e.g., the nucleon-Δ mass difference [13]. This spin-spin interaction must therefore also play a role when an external probe interacts with the three-quark baryon state. In the context of spin sum rules, the probe couples to the all possible axial currents in the nucleon. *That is, the probe not only senses a single quark current but a two-quark current as well*. The latter has an intermediate confined-quark propagator connecting the vertex of the probe and the spin-spin interaction between two quarks, and is similar to the exchange-current corrections which are well known in nuclear physics. In the case of the two-quark current, investigated in detail in Ref. [14] using the MIT bag model, the confined quark propagator was written as a sum over quark eigenmodes and the dominant contributions were found to come from the intermediate p-wave anti-quark states. The primary focus of Ref. [14] was however the one-gluon-exchange corrections to the magnetic moments and semi-leptonic decays of the baryon octet (see below).

Myhrer and Thomas [2] realized the importance of this correction to the flavor singlet axial charge and hence to the proton spin, finding that it reduced the fraction of the spin of the nucleon carried by quarks, calculated in the naive bag model by 0.15, i.e., $\Sigma \rightarrow \Sigma - 3G$ [2]. The correction term, G, is proportional to α_S times certain bag model matrix elements [14], where α_S is determined by the “bare" nucleon-Δ mass difference. Again, the spin lost by the quarks is compensated by orbital angular momentum of the quarks and anti-quarks (predominantly $\bar{u}$ in the p-wave).

The pion cloud

The pion cloud is an effective description of the quark-antiquark excitations which are required by the spontaneous breaking of chiral symmetry in QCD. In fact, describing a physical nucleon as having a pion cloud which interacts with the valence quarks of the quark core (the "bare" nucleon), in a manner dictated by the requirements of chiral symmetry, has been very successful in describing the properties of the nucleon [15, 16, 17]. The cloudy bag model (CBM) [15, 16] reflects this nucleon description where the nucleon consists of a bare nucleon, $|N>$, with a probability $Z \sim 1 - P_{N\pi} - P_{\Delta\pi} \sim 0.7$, in addition to being described as a nucleon and a pion and a Δ and a pion, with probabilities $P_{N\pi} \sim 0.20 - 0.25$ and $P_{\Delta\pi} \sim 0.05 - 0.10$, respectively. The phenomenological constraints on these probabilities were discussed, e.g. [18, 19].

The pion cloud effect on Σ was investigated early by Schreiber and Thomas [4], who wrote the corrections to the spin sum-rules for the proton and neutron explicitly in terms of the probabilities above. To summarize Ref. [4]: the pion cloud correction to the flavor singlet combination modifies the proton spin in the following manner:

$$\Sigma \to \left(Z - \frac{1}{3} P_{N\pi} + \frac{5}{3} P_{\Delta\pi} \right) \Sigma . \tag{1}$$

From the point of view of the spin problem, the critical feature of the pion cloud is that the coupling of the spin of the nucleon to the orbital angular momentum of the pion in the $N\pi$ Fock state favors a spin down nucleon and a pion with +1 unit of orbital angular momentum. This too has the effect of replacing quark spin by quark and anti-quark orbital angular momentum. Note that in the $\Delta\pi$ Fock component the spin of the baryon tends to point up (and the pion angular momentum down), thus enhancing the quark spin. Nevertheless, the wave function renormalization factor, Z, dominates, yielding a reduction by a factor between 0.7 and 0.8 for the range of probabilities quoted above.

Other "spin observables" affected by these corrections

Some of the quark hyperfine interaction (OGE) and the pion cloud corrections are illustrated in the following two Tables. A brief summary of these corrections are:

The effective OGE (i) This correction is vital in the understanding of the measured strength of $\Sigma^- \to n + e^- + \bar{\nu}_e$, see Table 2.
(ii) Essential to explain the magnetic moment inequality $|\mu_\Lambda| < |\mu_{\Xi^-}|$, Table 1,
(iii) OGE introduces configuration mixing in baryon-octet ground states which can affect strongly the radiative decay of excited baryons to the ground state baryons.

Pion cloud (i) Crucial component of the neutron charge distribution.
(ii) Provides a large iso-vector contribution to the nucleon magnetic moment.
(iii) Gives the leading non-analytic chiral-loop corrections to nucleon observables.

TABLE 1. The baryon magnetic moments from the valence quark, the pion cloud and the OGE contributions as evaluated in [14].

Baryon	Quark	Pion	OGE	Mag mom	PDG − 2002
μ_p	μ_q	$\delta\mu_\pi$	0	2.79	+2.79
μ_n	$-\frac{2}{3}\mu_q$	$-\delta\mu_\pi$	$\frac{2}{3}G'$	−1.92	−1.91
μ_{Σ^+}	$\frac{8}{9}\mu_q+\frac{1}{9}\mu_s$	$\frac{1}{2}\delta\mu_\pi^*$	0	2.46	$+2.458\pm0.010$
μ_{Σ^-}	$-\frac{4}{9}\mu_q+\frac{1}{9}\mu_s$	$-\frac{1}{2}\delta\mu_\pi^*$	$-\frac{2}{3}G'$	−1.20	-1.160 ± 0.025
μ_{Ξ^0}	$-\frac{2}{9}\mu_q-\frac{4}{9}\mu_s$	$\simeq 0$	$\frac{2}{3}G'$	−1.20	-1.250 ± 0.014
μ_{Ξ^-}	$\frac{1}{9}\mu_q-\frac{4}{9}\mu_s$	$\simeq 0$	$-\frac{2}{3}G'$	−0.73	-0.6507 ± 0.0025
μ_Λ	$-\frac{1}{3}\mu_s$	0	$\frac{1}{3}G'$	−0.61	−0.61

TABLE 2. The semi-leptonic decays of some baryons showing only the valence quarks and OGE contributions [14]. Other corrections are implicit.

$B' \to B$	Quark + OGE	PDG − 2002
$n \to p$	$\frac{5}{3}B+G\simeq 1.25$	1.27
$\Sigma^- \to n$	$-\frac{1}{3}B-2G\simeq -0.34$	−0.34
$\Lambda \to p$	$B \simeq 0.72$	0.72
$\Xi^- \to \Lambda$	$\frac{1}{3}B-G\simeq 0.19$	0.25 ± 0.05

Epilogue

When the pion cloud or gluon exchange corrections were first discussed, each one alone did not yield a correction large enough to resolve the “spin crisis". Furthermore, since the pion might contribute a substantial fraction of the observed mass-splitting between the N and Δ, to combine these two corrections would reduce the strength of OGE. However, in the last few years the chiral analysis of quenched and full lattice QCD calculations for the N and Δ masses as a function of quark mass [11], concluded that pion effects likely contribute 50 MeV or less of the 300 MeV $N-\Delta$ mass difference. We can therefore without too large an error combine the one-gluon-exchange and pion cloud corrections in the quark spin sum. This combined correction will give a Σ between 0.35 ($P_{N\pi}=0.25, P_{\Delta\pi}=0.05$) and 0.40 ($P_{N\pi}=0.20, P_{\Delta\pi}=0.10$) in excellent agreement with the modern data. We note that g_A is reproduced by the same corrections affecting the Σ. Relativity reduces the value of g_A from 5/3 to 1.09 and OGE and the pion cloud as well as the center-of-mass corrections will increase the g_A value from 1.09 to 1.27. As seen in Table 1 these corrections are cruzial in order to reproduce the baryon magnetic moments, i.e. the pion isovector cloud is an important correction to the nucleon magnetic moments and the OGE restores the ratio $\mu_p/\mu_n \simeq -3/2$![14].

We have used a model of confined quarks to compute the matrix elements of the axial current to find a Σ value relevant in the limit $Q^2 \to \infty$. Our result, $\Sigma \in (0.35, 0.40)$, agrees very well with the experimental value. The flavor singlet spin operator however has a non-zero anomalous dimension, γ, and the observable Σ should be renormalization group independent and gauge-invariant as defined by Larin *et al.* [20]. Motivated by the observation that a valence dominated quark model can only match experiment for parton distribution functions at a low scale, e.g. [21], our value of the quark spin would need to be multiplied by a non-perturbative factor involving the QCD β–function and γ. This evolution factor is truly non-perturbative and its three-loop perturbation theory evaluation by Larin *et al.* [20] is at best semi-quantitative.Nevertheless, it is rigorously less than unity and at three-loops gave a value of order 0.6–0.8 [22]. Multiplying the quark spin obtained above by this factor gives $\Sigma \in (0.21, 0.32)$, in excellent agreement with the current experimental value.

In conclusion, the impressive experimental progress aimed at resolving the spin problem has established that the quarks carry about 1/3 of proton's spin and that the gluonic contribution appears to be too small to account for the difference. Instead, well known nucleon structure features like the pion cloud, the quarks' hyperfine interaction, and the relativistic motion of the confined quarks, appear to explain the modern value of Σ. These new insights make us conclude that the missing spin should be accounted for by the orbital angular momentum of the quarks and anti-quarks – the latter associated with the pion cloud of the nucleon and the p-wave anti-quarks excited by the one-gluon-exchange hyperfine interaction. Exploring the angular momentum carried by quarks and anti-quarks is a major focus of the scientific program of the 12 GeV Upgrade at Jefferson Lab., and is a promising way to test the model ideas present here.

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REFERENCES

1. J. Ashman *et al.* [European Muon Collaboration], *Phys. Letters B* **206**, 364 (1988).
2. F. Myhrer and A.W Thomas, *Phys. Rev. D* **38**, 1633 (1988)
3. H. Høgaasen and F. Myhrer, *Phys. Letters B* **214**, 123 (1988)
4. A.W. Schreiber and A.W. Thomas, *Phys. Letters B* **215**, 141 (1988)
5. A.V. Efremov and O.V. Teryaeb, Dubna report *JINR-E2-88-287* (1988)
6. E. Leader and M. Anselmino, *Z. Phys. C* **41**, 239 (1988); *AIP Conf. Proc.* **187**, 764 (1989).
7. G. Altarelli and G.G. Ross, *Phys. Letters B* **212**, 391 (1988)
8. R.D. Carlitz, J.C. Collins and A.H. Mueller, *Phys. Letters B* **214**, 229 (1988)
9. See these proceedings for COMPASS, HERMES, PHENIX and STAR collab. contributions.
10. F. Myhrer and A. W. Thomas, *Phys. Letters B* **663**, 302 (2008);
    A.W. Thomas, preprint arXiv:0803.2775 [hep-ph].
11. R. D. Young *et al.* Phys. Rev. D **66**, 094507 (2002); *Nucl. Phys. Proc. Suppl.* **109A**, 55 (2002).
12. I.C. Cloet, W. Bentz and A.W. Thomas, *Phys. Letters B* **659**, 214 (2008)
13. A. De Rujula *et al.* Phys. Rev. D **12**, 147 (1975); A. Chodos *et al.* Phys. Rev. D **10**, 2599 (1974).
14. H. Høgaasen and F. Myhrer, Phys. Rev. D **37**, 1950 (1988).
15. S. Theberge, A. W. Thomas and G. A. Miller, Phys. Rev. D **22**, 2838 (1980)
16. A. W. Thomas, Adv. Nucl. Phys. **13**, 1 (1984).
17. F. Myhrer, G.E. Brown and Z. Xu, Nucl. Phys. A **362**, 317 (1981);
18. J. Speth and A. W. Thomas, Adv. Nucl. Phys. **24**, 83 (1997).
19. W. Melnitchouk, J. Speth and A. W. Thomas, Phys. Rev. D **59**, 014033 (1999)
20. S. A. Larin, Phys. Lett. B **334**, 192 (1994); S. A. Larin *et al.* Phys. Lett. B **404** , 153 (1997).
21. M. Gluck and E. Reya, Z. Phys. C **43**, 679 (1989)
22. H. Høgaasen and F. Myhrer, Z. Phys. C **68**, 625 (1995)[arXiv:hep-ph/9501414].